ESTIMATING PREFERENCES TOWARD RISK:

EVIDENCE FROM DOW JONES

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ABSTRACT

What do investor utility functions look like? We show how returns on a stock and prices of call options written on that stock can be used jointly to recover utility of wealth function of the marginal investor in the stock. We study whether non-standard preferences have an impact sufficiently large that it is present in the stock prices. Using options on the stocks in the Dow Jones Index, we show support for non-concave utility functions with reference points proposed by Kahneman and Tversky, Friedman and Savage, and Markowitz. The evidence for Kahneman and Tversky Prospect Theory value function, and Friedman and Savage and Markowitz utility functions is much stronger than the support for the standard concave utility function. Together the utility functions with convex regions and with reference points account for 80% of the market capitalization of the sample stocks. This is the first study to report findings of these utility functions using the prices of individual stocks (non-experimental data). We also investigate a closely related question of whether different assets reflect different risk preferences. We find evidence showing that different stocks reflect different types of investor utility function.

Keywords: Utility function; Investor risk preferences; Risk seeking; Reference-Dependent Preferences

JEL Codes: D01, G11, G12

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Introduction

Rabin and Thaler (2001) argue that the evidence against the expected utility maximization paradigm based on a classical risk-averse utility function is overwhelming. Standard models have difficulty explaining the first moment (equity premium) and the second moment (excess volatility) of asset returns.[1] Rational models are also under increasing attack after the boom in U.S. stock prices in the late 1990's, often referred to as one of the biggest bubbles in financial history. Many financial economists, in the attempt to understand price behavior, have focused on behavioral explanations.[2] Based on this growing amount of evidence, Rabin and Thaler conclude that we, as financial economists, need to "concentrate our energies on the important task of developing better descriptive models of choice under uncertainty."

A better understanding of investor preferences will help explain the behavior of risk premia both in the cross-section and in the time series. But what utility function should we use?

In this paper we let the data speak. We use market data for each individual stock in the *Dow Jones Industrial Average* to determine the shape of the investor's utility function for that stock. The recovered shapes are then compared to the previously hypothesized utility functions. We show that asset prices reflect utility functions that contain risk-averse and risk-seeking regions. The support for utility functions with convex regions and with reference

[1] See Constantinides (2002) for a detailed discussion. Constantinides (2002) concludes that the evidence does not support the case for abandoning the rational economic model. Benartzi and Thaler (1995) review the evidence regarding the equity premium puzzle—the observation by Mehra and Prescott (1985) that the combination of a high equity premium, a low risk-free rate, and a smooth consumption is difficult to explain with plausible levels of investor risk aversion.

[2] Shiller (2000) attributes the boom in technology stocks to irrational exuberance. Several authors show how speculative bubbles may be caused by investor overconfidence: Scheinkman and Xiong (2003), Hong, Scheinkman and Wie (2004), and Sornette and Zhou (2003).

points—as postulated by Kahneman and Tversky, Friedman and Savage, and Markowitz—is much stronger that the evidence in favor of the standard uniformly risk-averse preferences.

This is a significant finding. Kahneman and Tversky, Friedman and Savage, and Markowitz developed their utility functions to explain very general patterns in human behavior. Their purpose was not to explain specific well-documented patterns in asset prices. However, we find that these preferences are reflected in the prices of *Dow Jones* stocks. Individual behavior is not averaged out or arbitraged away. In addition, the evidence is strong. These three types of utility functions account for 80% of the market capitalization of the sample stocks.

Modifications to the standard concave utility function are a formal way of explaining individual behavior within a decision theory under risk. The modifications date back to the work of Friedman and Savage (1948) who, in order to construct a rational explanation for the coexistence of gambling and owning insurance in human behavior, propose that an individual's utility of wealth function is composed of two (strictly) concave segments separated by a (strictly) convex segment (Figure 1). Concavity implies risk-averse behavior and convexity implies risk-seeking behavior. For investors with such a utility function simultaneous purchase of insurance and lotteries is fully rational. Markowitz (1952) argues that the Friedman and Savage utility function should be modified so that the inflection point where the concave region turns into the convex region is located exactly at the individual's current wealth (Figure 2), thereby introducing a *reference point* into the utility function.

Perhaps the most well-known class of value function is the prospect theory *S*-shaped function suggested by Kahneman and Tversky (1979, 1992). Based on their experimental results, they suggest that the value function is convex in the domain of losses (below the current wealth level) and concave in the domain of gains (above the current wealth). This function changes from risk-seeking to risk aversion at the current level of wealth. The function captures *loss aversion*, the empirically demonstrated tendency for people to weight losses significantly more heavily than gains.

In the neighborhood of the current wealth, the utility functions proposed by Kahneman and Tversky and by Markowitz have the opposite behavior. The former describes investors as being risk-seeking over losses and risk-averse over gains, while the latter describes investors as being risk-averse over losses and risk-seeking over gains. For both functions, the convex regions are consistent with risk-seeking behavior, a feature absent from classic models.[3] Both functions explicitly postulate the existence of a reference point—the current wealth.

More recently, non-standard utility functions have been used to explain asset pricing anomalies.[4] Bernartzi and Thaler (1995) use Kahneman and Tversky's (1979) theory of *loss aversion*, combined with a short evaluation period of individuals (*myopia*) to explain the equity premium puzzle at the *aggregate* stock market level.[5] Loss aversion refers to the tendency of individuals to be more sensitive to reductions in their levels of wealth than to increases. Estimates of loss aversion are typically close to 2.0: losses hurt roughly twice as much as gains yield pleasure.[6] Bernartzi and Thaler (1995) analyze the portfolio problem of a loss averse investor who allocates wealth between T-bills and the stock market. They find that

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[3] Several authors have shown how Friedman-Savage utility can arise within a fully rational utility maximization framework. We review the literature on convexity in the theory of choice in the Appendix, where we also discuss that convexities in individual utility functions do not necessarily contradict the capital market equilibrium, or even a large aggregate risk premium.

[4] The new theories are driven by the desire to explain the known patterns in asset returns from first principles. Theoretical models that are based on modifying standard preference structures include Constantinides (1990), Barberis, Huang, and Santos (2001), and Campbell and Cochrane (1999). Grinblatt and Han (2004) develop a pricing model in which a group of investors is subject to Prospect Theory and Mental Accounting behavior. Shefrin and Statman (2000) build a behavioral portfolio theory (BPT). The optimal portfolios of BPT investors resemble combinations of bonds and lottery tickets, consistent with Friedman and Savage's (1948) observation.

[5] Thaler, Tversky, Kahneman, and Schwartz (1997) present experimental evidence on myopic loss aversion.

[6] These estimates come both from risky choice (Tversky and Kahneman 1992) and from riskless choice (Kahneman, Knetsch, and Thaler 1990).

the investor is reluctant to allocate much to stocks, even if the expected return on the stock market is set equal to its high historical value. The size of the equity premium is consistent with the previously estimated parameters of prospect theory. Coval and Shumway (2005) find that market makers exhibit loss aversion. Market makers take above average risk after experiencing losses earlier that same day. This risk-seeking behavior has a short-term affect on prices. Barberis and Huang (2004) use Kahneman and Tversky's (1992) cumulative prospect theory to explain the underpricing of IPOs. In a theory developed by Brunnermeier and Parker (2005) agents can be risk-loving when investing in assets with skewed payoffs and at the same time risk averse when investing in non-skewed assets.

The challenge remains to show that non-standard utility functions have an impact on asset prices sufficiently large that it can be detected empirically. On the one hand, risk-seeking, loss aversion, and the existence of a point of reference all may affect prices—and should be detected in prices—because they affect saving and investment behavior. For example, loss aversion affects savings because once households get used to a particular level of disposable income, they tend to view reductions in that level as a loss (Thaler and Benartzi 2004). On the other hand, there are several reasons why asset prices may not reflect biases. It may be the case that behavioral biases of individual investors vanish in the aggregate. Different investors may have different biases. In equilibrium when there are sufficiently many investors these biases may off-set and cancel each other out. The impact of behavioral biases on asset prices may also be difficult to detect because rational arbitrageurs may be at work, arbitraging the biases away. Coval and Shumway (2005), for example, find that prices set by loss averse individuals are reversed more quickly than prices set by other unbiased traders. In this case it appears that the market is able to distinguish the risk-seeking behavior. However, if the rational arbitrage forces have limits then prices will reflect biases.[7] There is another conceptual difficulty recently discovered by several researchers. Surprisingly, theories based on very different assumptions may lead to remarkably similar results. For example, mean-variance analysis (MV) is based on the assumption of utility maximization by a risk-averse investor. In contrast, in Prospect Theory (PT) risk aversion does not globally prevail – individuals are risk-seeking regarding losses. Also, PT investors make decisions based on *change* of wealth rather than the *total* wealth. Levy and Levy (2004) show that, counter-intuitively, when diversification between assets is allowed, the MV and PT-efficient sets almost coincide. If, as the authors conclude, one may employ the MV optimization algorithm to construct PT-efficient portfolios, then portfolio composition, asset demand, and resulting asset prices may look very similar in PT and MV cases. This may make it difficult to detect the impact of PT preferences using stock prices.

Investor risk preferences are at the center of the debate between rational and behavioral economists. These preferences are the benchmark which researchers use to distinguish fully rational behavior from various behavioral biases. Yet, the empirical studies of the shape of investor preferences toward risk are rare. Most closely related to our work is the study by Post and Levy (2005). The authors use various stochastic dominance criteria that account for (local) risk seeking. They study the efficiency of the market portfolio relative to benchmark portfolios formed on market capitalization, book-to-market-value, and price momentum. They find some evidence supporting utility functions with risk aversion for losses and risk seeking for gains in aggregate stock returns.

We take a step forward in this line of research by using individual stock market data to determine the shape of the investor's utility function. We estimate risk aversion as a

[7] See, e.g. DeLong et al. (1990), Shleifer and Vishny (1997), and Daniel et al. (2001).

function of wealth using option and stock prices for all stocks comprising the *Dow Jones Industrial Index*.[8] Using the Arrow-Pratt definition of risk aversion, we are then able to determine the utility function implied by risk aversion. The focus of this paper is on non-experimental measurement of utility. Our approach is flexible. We do not assume any particular function and we do not rely on a specific option pricing model when working with the options data.[9] This is the first paper to show support for several different well-known utility functions in the prices of individual stocks and option contracts written on these stocks.

By working with individual stocks we can study whether prices of different securities reflect different risk preferences. If the same marginal investor (a representative investor) prices all assets in the economy and this investor has a canonical risk averse utility function, then we should expect the same risk aversion estimates for all stocks. Our results, however, indicate support for a variety of utility functions. We find evidence to support the existence of Constant Absolute Risk Aversion (CARA), Constant Relative Risk Aversion (CRRA), Friedman and Savage, Markowitz, and Kahneman and Tversky utility functions. This is the first study to find evidence supporting Markowitz and Friedman-Savage utility functions in stock prices. Even more striking is the existence of seemingly opposite behavior. Kahneman-Tversky's S-shaped utility function and Markowitz utility function are complete opposites

[8] We discuss the modifications we make to the methodologies of Jackwerth (2000) and Bliss and Panigirtzoglou (2004) later in the paper.

[9] The method allows us to examine the shape of the investor utility function. One more issue can be addressed with this approach. In the canonical asset pricing theory the same investor – the marginal investor – sets the prices of all stocks. If there is a clientele effect where different types of investors purchase different securities, then it may be the case that prices of different stocks reflect a variety of preferences. Because we do not rely on a specific option pricing model, we do not need to explain the behavior of option prices within a model. In other words, we are not "fitting a smile."

around the current wealth level. The first exhibits risk-loving behavior over losses and risk-aversion over gains, whereas the latter describes risk aversion for the region of losses (immediately below current wealth) and risk-loving for the region of gains (immediately above current wealth). To illustrate our findings, Figure 3 shows risk aversion (as a function of wealth) of *Walt Disney* (DIS) investors, a company with capitalization in excess of $47 Billion and one of the stocks in the *Dow Jones Industrial Average*. Risk aversion is positive over losses then changes to negative over gains. This is consistent with the Markowitz utility function.

The total market capitalization of the firms in the sample at the end of the sample period (December 31, 2003) equals $3,882.5 billion. Of the 41 firms considered, thirty-four firms have risk aversion profile that includes both risk averse and risk-seeking regions, consistent with the hypotheses of Kahneman and Tversky, Friedman and Savage, and Markowitz. The total market capitalization of these companies equals $3,071 billion or 80% of the sample total. The remaining 20% of the market capitalization are distributed among different classes of utility functions as follows. Two firms with total capitalization of $144.3 billion (3.7% of the total) fit the profile of CARA preferences and one firm with the market capitalization of $44.6 billion is classified as CRRA. The risk aversion profiles for four firms do not fit in any of the above categories. These are the firms that we assign to the "other" category. Their combined market capitalization is $622.7 billion, or 16% of the total.

These findings have several implications to asset pricing research. Friedman-Savage, Markowitz, and Kahneman-Tversky utility functions were developed to explain individual behavior. By finding patterns of risk aversion consistent with these functions, we show that individual preferences toward risk may have an impact on asset prices. There is another dimension to our findings. We show that different utility functions characterize marginal investor for different assets (stocks). We show this with a new methodology and in a new statistical setting. Lastly, the evidence supporting different marginal investors for different securities challenges the representative agent paradigm. The evidence is consistent with investor behavior where stocks with different characteristics attract different types of investors. This notion is consistent, for example, with style investing (Barberis and Shleifer 2003). In the style investing paradigm investors classify stocks into various styles (for example, value stocks and growth stocks) and take the style classification into account when allocating funds. If style investing is taking place then investors with different preferences will gravitate to their preferred styles, changing the demand and affecting the prices. It then may be the case that prices of assets that are classified into different styles will reflect the differences in preferences.

Several authors have pointed out that it is difficult, from observing stock prices alone, to differentiate competing theories that attempt to explain price patterns. This difficulty is the central argument in Brav and Heaton (2002). The authors study two competing theories of financial anomalies: "behavioral" theories built on investor irrationality, and "rational structural uncertainty" theories built on incomplete information about the structure of the economic environment. The authors conclude that although the theories relax opposite assumptions of the rational expectations ideal, their mathematical and predictive similarities make them difficult to distinguish. We show that by using options, it is, to some extent, possible to overcome these difficulties and to recover investor preferences toward risk. We now develop the methodology more formally.

Methodology

We show how returns on an asset (a stock) and prices of options written on that asset can be jointly used to recover utility function of investors in the stock. We develop the methodology in this section.

The method for recovering utility functions from observed asset prices consists of two steps. First, we use returns on a stock and prices of options on that stock to estimate the Arrow-Pratt coefficient of risk aversion. Second, we use the estimated coefficient of risk aversion to recover the utility of wealth function.

We discuss the second step first. We show that if the coefficient of risk aversion as a function of wealth is known, then it is possible to recover the utility of wealth function. Let $u(w)$ be a twice continuously differentiable utility of wealth function. The Arrow-Pratt absolute risk aversion is defined as

$$RA(w) \equiv -\frac{u''(w)}{u'(w)}. \tag{1}$$

Relative risk aversion is defined as $RR(w) \equiv w \cdot RA(w)$. Suppose we have constructed an estimate of the risk aversion as a function of wealth $\hat{a}(w) \approx RA(w)$. Then, using the definition of risk aversion, the utility function is a solution to the ordinary differential equation,

$$-\frac{u''(w)}{u'(w)} = \hat{a}(w). \tag{2}$$

For example if the estimated risk aversion coefficient is a constant, $\hat{a}(w) = K$, then the differential equation becomes

$$-\frac{u''(w)}{u'(w)} = K.$$

The solution is the familiar negative exponential utility function (with constant absolute risk aversion, or CARA),

$$u(w) = -\frac{C_1}{a} e^{-a \cdot w} + C_2,$$

where $C_1, C_2 \in \Re$ are the constants of integration.

Another familiar example is the power utility function (CRRA). If the estimated coefficient of absolute risk aversion is a function of wealth, $\hat{a}(w) = (1-\gamma)/w$, then the relative risk aversion is constant

$$\hat{R}(w) = w \cdot \hat{a}(w) = 1 - \gamma .$$

The ODE in this case is

$$-\frac{u''(w)}{u'(w)} = \frac{1-\gamma}{w}$$

and the solution is the power utility function

$$u(w) = C_1 \frac{W^{\gamma}}{\gamma} + C_2,$$

where $C_1, C_2 \in \Re$ are the constants of integration.

The S-shaped Prospect Theory value function suggested by Kahneman and Tversky (1979, 1992) is one of the most well-known and most investigated in finance research. Utility is defined over gains and losses. This function is characterized by one inflection point located at the individual's current wealth. The individual is risk seeking for losses (the function is convex below the inflection point) and is risk averse for gains (the function is concave above the inflection point). Specifically, Kahneman and Tversky propose a value function of the following form:

$$v(x) = \begin{cases} x^{\alpha}, & x > 0 \\ -\lambda(-x)^{\beta} & x < 0 \end{cases}$$

They estimate parameter values as $\alpha = \beta = 0.88$ and $\lambda = 2.25$.

To see that this function implies risk-seeking in the domain of losses, $x < 0$, compute the first and the second derivatives in the domain of losses. Indeed, $dv(x)/dx = \beta\lambda(-x)^{\beta-1}$ and $d^2x/dx^2 = -\beta(\beta-1)\lambda(-x)^{\beta-2}$. By definition of risk aversion, $RA = -u''(x)/u'(x)$ and in this case $RA = (1-\beta)/x$, which is negative (indicating risk-seeking) for $\beta < 1$ and $x < 0$.

Let $w = W/W_0$ be normalized wealth (W_0 is the current wealth). Suppose that a researcher observes an empirical risk aversion function that is negative below the initial wealth and changes sign at the initial wealth to become positive above it,

$$\hat{a}(w) = \begin{cases} < 0, & if \quad w < 1, \\ = 0, & if \quad w = 1 \\ > 0, & if \quad w > 1 \end{cases}$$

This finding is evidence in support of Kahneman and Tversky value function. To recover the utility function we can solve ODE (2) numerically. Since risk aversion is estimated from a data sample, the statements about the sign of the risk aversion coefficient and about the equality with zero are statistical statements. To claim support for Kahneman-Tversky utility we must statistically show that $\hat{a}(w) < 0$ in a region below $w = 1$, $\hat{a}(w) = 0$ at $w = 1$, and $\hat{a}(w) > 0$ in the region above $w = 1$.

The Friedman-Savage utility function also contains both concave and convex regions. Such a utility function can be described by a convex, quadratic empirical risk aversion function $\hat{a}(w)$ with two positive roots (this is a sufficient, but not necessary condition for Friedman-Savage utility).[10] The utility function corresponding to such risk

[10] An example of such risk aversion function, estimated for Chevron (CHV) is shown in Figure 4. The pattern will be verified by statistical tests.

aversion profile is characterized by two concave regions separated by a convex region. Suppose the estimated risk aversion is a quadratic function,

$$\hat{a}(w) = Aw^2 + Bw + C\ ,$$

then the utility function $u(w)$ is determined from the ordinary differential equation,

$$-\frac{u''(w)}{u'(w)} = Aw^2 + Bw + C\ .$$

The solution to this equation in general form is given by

$$u(w) = C_2 + C_1 \int_{\zeta}^{w} Exp\left[-\frac{At^3}{3} - \frac{Bt^2}{2} - Ct \right] dt,$$

where ζ is a dummy variable, and $C_1, C_2 \in \Re$.

Markowitz (1952) suggests that the Friedman and Savage utility function should be modified so that the inflection point where the concave region turns into the convex region is located exactly at the current wealth. Markowitz utility function has *three* inflection points (Figure 2). The middle inflection point is defined to be at the current level of wealth.

The following pattern observed in the empirical risk aversion is consistent with Markowitz utility function. Empirical risk aversion is first negative, then positive immediately below the initial wealth, and changes to negative at initial wealth,

$$\hat{a}(w) = \begin{cases} < 0, & if \quad w < D \\ > 0, & if \quad D < w < 1 \\ = 0, & if \quad w = 1 \\ < 0, & if \quad 1 < w < E \\ > 0, & if \quad E < w \end{cases},$$

where D<1 is the first inflection point (below current wealth), and E>1 is the third inflection point above the current wealth. An empirical risk aversion function with this shape is evidence consistent with Markowitz utility function. To recover the utility function, we can solve ODE (2) numerically. Since risk aversion is computed from a data sample, the statements about the sign of the risk aversion coefficient and about the equality with zero are statistical statements. The statistical hypotheses are whether $\hat{a}(w) < 0$ in a region below some *D*<1, positive in the region between *D* and $w = 1$, $\hat{a}(1) = 0$, negative in the region between 1 and *E*, and $\hat{a}(w) > 0$ in the region above the last inflection point *E*. One caveat. Markowitz utility function has a complex shape. Neither Markowitz nor Friedman and Savage state where the inflection points are located with the exception of Markowitz utility changing from concave to convex at the current wealth level. Therefore, we may not have enough data to recover this complex shape in full. We observe only a section of the risk aversion function, away from very large gains or losses. The Markowitz function, however, has a distinct shape near the current wealth. By observing risk aversion near $w = 1$, we can capture a very interesting and unique characteristic of this utility function.

In the neighborhood of the current wealth, the utility functions proposed by Kahneman and Tversky and by Markowitz have the opposite behavior. The first changes from convex to concave, the second changes from concave to convex. This gives an opportunity to test whether the data support one or the other. If, in the neighborhood of current wealth, estimated risk aversion changes from negative to positive, then this is evidence in favor of Kahneman and Tversky. If, on the other hand, risk aversion changes from positive to negative, then the data supports Markowitz utility function.

To summarize, we will study the following utility functions:

1. Constant Absolute Risk Aversion: The function is concave throughout. Estimated absolute risk aversion coefficient is positive and independent of wealth.

2. Constant Relative Risk Aversion: The function is concave throughout. The estimated coefficient of absolute risk aversion is positive and is a function of wealth, $\hat{a}(w) = (1-\gamma)/w$.

3. Kahneman and Tversky. The utility function is convex in the domain of losses (below current wealth) and concave in the domain of gains (above current wealth). The estimated coefficient of risk aversion is negative below current wealth, equals zero at current wealth, and then positive above current wealth.

4. Friedman and Savage Utility Function. The function is concave, convex, and then concave again. Estimated coefficient of risk aversion is positive, negative, and then positive.

5. Markowitz Utility Function. The function consists of four regions: convex, concave, convex, and concave. In the neighborhood of the current wealth the function changes shape from concave to convex. The estimated coefficient of risk aversion is positive (immediately) below current wealth, equals zero at current wealth, and is then negative immediately above current wealth.

We have established a connection between the coefficient of risk aversion as a function of wealth and the shapes of utility functions proposed by several researchers. We have shown that if risk aversion is estimated than we can construct an estimate of the utility function by solving a second order ODE. We now discuss how the marginal investor's risk aversion function for a particular stock can be estimated from the data on stock returns and options written on the stock.

Recovering Risk Aversion

An estimate of risk aversion can be obtained from asset prices. There is a relationship between the risk-neutral probability distribution of returns on a stock i, $P(S_{i,T})$, subjective (true) probability distribution $Q(S_{i,T})$, and investor risk aversion. The relation is,[11]

$$Risk\ Aversion = -\frac{U''(S_{i,T})}{U'(S_{i,T})} = \frac{Q'(S_{i,T})}{Q(S_{i,T})} - \frac{P'(S_{i,T})}{P(S_{i,T})},$$

where $U(\bullet)$ is the investor's time–separable utility of wealth. Therefore, knowing the subjective distribution and the risk neutral distribution is sufficient to find risk aversion.

To determine the two distributions, we combine the methodologies of Bliss and Panigirtzoglou (2002, 2004) and Jackwerth (2000). Using option prices for a particular underlying stock, we estimate the risk-neutral probability density function (PDF) according to Bliss and Panigirtzoglou (2002, 2004). We then use five years of past monthly stock returns to determine a risk-adjusted (or, subjective) PDF using a nonparametric kernel density estimator similar to the one used in Jackwerth (2000). Risk aversion is the adjustment required to transform the risk-neutral PDF into the risk-adjusted PDF. Using

[11] See, for example, discussion in Jackwerth (2000).

this method, the risk aversion coefficient can be estimated for every trading day for any asset for which option prices are available.

Risk-neutral probability distribution

One method for finding the monthly risk-neutral distribution is proposed in Jackwerth and Rubinstein (1996). The method is based on a search for the smoothest risk-neutral distribution, which at the same time explains the option prices. The trade-off between the two contradicting goals is exogenously specified. Three problems arise with this approach (Jackwerth 2000). First, matching the option prices by minimizing the squared error puts more weight on in-the-money compared to out-of-the-money options. Second, the Jackwerth-Rubinstein method does not account for the fact that at-the-money option prices vary less throughout the day than away-from-the-money options. Third, the Jackwerth-Rubinstein method uses as a measure of smoothness the integral of squared curvature of the probability distribution.

We use a different approach. We know from option pricing theory that the risk-neutral PDF is embedded in option prices. Let T be the expiration date of an option. The PDF, $f(S_{i,T})$, for the underlying asset i at time T has been shown to be related to the price of the European call option, $C(S_{i,t}, K, t)$, by Breeden and Litzenberger (1978). Here, K is the option strike price and $S_{i,t}$ is the price of underlying i at time t where $t<T$. This relationship is

$$f(S_{i,T}) = e^{r(T-t)} \frac{\partial^2 C(S_{i,T}, K, t)}{\partial K^2}\bigg|_{K=S_{i,T}} .$$

For each underlying asset, i, and for each expiration date, however, the function $C(S_{i,t}, K, t)$ is unknown and only a limited set of call options with different strike prices exist. Therefore, in order to calculate the second derivative we estimate a smoothing function using option prices with different strike prices but with the same expiration dates.

Instead of estimating such a smoothing function in option price/strike price space, we follow Bliss and Panigirtzoglou (2002, 2004) by first mapping each option price/strike price pair to the corresponding implied volatility/delta. We fit a curve connecting the implied volatility/delta pairs using a weighted cubic spline where the option's vega is used as the weight. We take 300 points along the curve and transform them back to the option price/strike price space. We thus obtain a smoothed price function, which we numerically differentiate to produce the estimated PDF. Bliss and Panigirtzoglou (2002) find that this method of estimating the implied volatility smile and the implied PDF is "remarkably free of computational problems."

A weighted natural spline is used to fit a smoothing function to the transformed raw data. The natural spline minimizes the following function:

$$\min_{\theta}\sum_{j=1}^{N} w_j\left(IV_j - IV\left(\Delta_j,\theta\right)\right)^2 + \lambda\int_{-\infty}^{\infty} g''\left(x;\theta\right)^2 dx,$$

where we omit the company-identifying index, i, for brevity; IV_j is the implied volatility of the j^{th} option on stock i in the cross section; $IV\left(\Delta_j,\theta\right)$ is the fitted implied volatility which is a function of the j^{th} option delta, Δ_j, and the parameters, θ, that define the smoothing spline, $g\left(x;\theta\right)$; and w_j is the weight applied to the j^{th} option's squared fitted implied volatility error. Following Bliss and Panigirtzoglou (2004), in this paper we use the option vegas, $v \equiv \partial C / \partial \sigma$, to weight the observations. The parameter λ is a *smoothing parameter* that controls the tradeoff between goodness-of-fit of the fitted spline and its smoothness measured by the integrated squared second derivative of the implied volatility function.

From the estimated cubic spline curve, we take 300 equally spaced deltas and their corresponding implied volatilities and transform them back to option price/strike price space using the Black-Scholes option pricing formula that accounts for dividends paid on the stock. However, although the deltas are equally spaced, the strike prices that are obtained after the conversion are not. We use a cubic spline for a second time to fit a curve connecting the 300 unequally spaced call price/strike price pairs. This allows us to choose 300 equally spaced strike prices with their corresponding call prices. Finally, we use finite differences to estimate the second derivative of the call price with respect to the strike price. This yields the risk-neutral PDF. This procedure does not depend on a specific option pricing model (Bliss and Panigirtzoglou 2004).

Subjective probability distributions

We use a kernel density estimator to estimate the subjective (risk-adjusted) probability density functions. Similar procedure is used in Jackwerth (2000).[12] We use the most recent 60 months of stock return data to estimate the risk-adjusted distribution. To find estimates for January 1996, we use monthly return data from January 1991 to December 1995. All information used in the calculation is part of the investors' information set. Other windows were considered but results were highly correlated. For example, we tried a window of past returns with a lag of one year or six month, and we tried using 72 months of returns instead of 60. Varying our initial choices does not change the results.

[12] This is different from Bliss and Panigirtzoglou (2004) who first hypothesize a utility function (power and exponential utility) for the investor and then use this function to convert the risk-neutral PDF to the subjective PDF. We do not follow this approach because we do not hypothesize a utility function.

We calculate monthly non-overlapping returns from our 5-year sample and compute the kernel density with a Gaussian kernel. The bandwidth

$$h = \hat{\sigma}\left[4/(3n)\right]^{1/5},$$

where h is the kernel bandwidth, $\hat{\sigma}$ is the standard deviation of the sample returns, and n is the number of observations, is selected by recommendation of Jones, Marron and Sheather (1996).

Data

The data for this study consists of daily closing prices of call options written on the stocks that are included in the *Dow Jones Industrial Average*. The study covers the eight-year period from January 2, 1996 through December 31, 2003, since this is the period when prices of options on individual stocks are available to us. In addition to the daily closing option prices, we use monthly stock returns and daily stock closing prices from CRSP. Table 1 lists the firms in the sample. We include all stocks that were a Dow component at some time during this period. For example, *Microsoft* and *Intel* were added to the Dow in November, 1999 but we use the data back to the beginning of 1996. *Goodyear* was removed from the Dow in November, 1999 but we use the data for this stock through the end of the sample. There are cases when there is not enough options data to estimate risk aversion for a stock. We exclude *Bethlehem Steel* (BS), *Westinghouse Electric* (WX), and *Woolworth* (Z) because of insufficient data. We begin with 44 firms, and include 41 firms in the final study.

Summary statistics for the firms in the sample is given in Table 2. The table gives market capitalization at the beginning and the end of the sample, the total holding period return over the sample period (HPR), as well as risk and return characteristics of the stocks in the sample. *General Electric* (GE) is the largest firm in the sample with market capitalization of $122 Billion and *AT&T* (T) is the second largest with market capitalization of $107 Billion. At the end of our sample, GE is the largest firm in the sample with market capitalization of $311 Billion (Microsoft is the second largest with $295 Billion). The average market capitalization has grown from $36.9 Billion at the beginning to $99.6 Billion at the end of the sample period. *Citigroup* (C) is the best performing firm in the group, with HPR of 1,181%. *Pfizer* (PFE) has the second highest HPR of 580%. The average HPR is 170%, which corresponds to 2.94% return compounded annually over the period of 8 years. Table 2 also displays risk and return characteristics of the firms in the sample, including firm betas (computed both with respect to S&P 500 and CRSP Value-Weighted Index), average monthly returns, and variances of returns.

To estimate risk aversion we need prices of options written on the stocks in the sample. All previous researchers have studied risk aversion using the options on the S&P 500 Index. For index options, a relatively large cross section of strikes exists, all with the same expiration date. Because of this large selection of options, Bliss and Panigirtzoglou (2004) require at least 5 such options in order to do their estimation. We are considering individual stocks. Companies tend to have a smaller cross section of options with different strike prices and their options are comparatively less liquid than index options. Because of this, we require a firm to have options with at least three different strike prices. In addition, similar to Jackwerth (2000), we estimate risk aversion with a constraint on the money-ness. Jackwerth only considers options such that the ratio of the strike price to the stock price is between 0.84 and 1.12. This procedure eliminates far-away-from-the-money observations. This may cause a problem of missing observations, but only when there are large movements in the stock price. Since options with only a few different strikes are traded for each firm (usually five or six strikes), a large enough movement in the stock price causes the money-ness to fall in a window that does not have three option contracts for us to use. We do not estimate risk aversion for such days. Not surprising, tossing out options that are way in the money or way out of the money affects risk aversion estimates in the tales of the distribution. Our robustness checks indicate that the estimates in the middle of the distribution are generally unaffected by the money-ness constraints. We find our results to be robust to the selection of options.

For our estimation, we consider options that expire between one and four months from day *t*. Options on stocks generally exist with expiration dates at three-month intervals.[13] For example, options on Microsoft expire in January, April, July, and October. Therefore, for all days in January, we use options expiring in April. For all days in March, we use those options expiring in July. We use this approach to maintain a relatively constant horizon for our analysis, and at the same time to have a sufficient number of option contracts to obtain reliable risk aversion estimates.

For each of the 41 stocks in the sample, for each trading day between January 4, 1996 and December 31, 2003 we calculate estimates of Arrow-Pratt risk aversion functions across wealth, a computationally intensive process. The wealth lies in the interval [0.95, 1.05]. For each wealth level we calculate the mean risk aversion across the period 4-Jan-96 – 31-Dec-03. In addition, we calculate their empirical standard deviations.

Results

After estimating risk aversion for the firms in the sample, we classify the risk aversion patterns as being consistent with several classes of utility functions. We find evidence supporting the existence of five distinct utility functions: Kahneman and Tversky,

[13] See Battalio, Hatch, and Jennings (2004), and Mayhew and Mihov (2004) for the description of the equity options markets including institutional background.

Friedman and Savage, Markowitz, Constant Absolute Risk Aversion (CARA) and Constant Relative Risk Aversion (CRRA). Our final classification is listed in Table 3.

Evidence supporting the utility function with risk-seeking regions is extensive. Figure 4 displays estimated risk aversion for four firms that support Friedman and Savage utility function: *Chevron* (CHV), *General Electric* (GE), *General Motors* (GM) and *Proctor and Gamble* (PG).

More surprising is the support for the Markowitz utility function. This function is characterized by the restrictive condition requiring risk aversion to change from positive to negative at current wealth, $w=1$. Figure 3 displays estimated risk aversion as a function of wealth for four firms supporting the existence of the Markowitz utility function: *3M Company* (MMM), *Eastman Kodak* (EK), *Walt Disney* (DIS), and *PepsiCo* (PEP).

Our classification consists of several steps. We first identify risk aversion functions having both positive and negative regions. Such risk aversion functions characterize Friedman and Savage, Markowitz, and Kahneman and Tversky utility functions but do not characterize CARA and CRRA utility functions. For each firm, using daily risk aversion estimates from 1996 to 2003 and wealth levels from 0.95 to 1.05 with step size 0.001, we test whether the median and mean risk aversion for each wealth level is significantly different from zero. We use a sign test to determine if the median is significantly different from zero and we use the Wilcoxon Signed Rank Test to measure whether the mean is significantly different from zero. We classify the risk aversion for a particular wealth level as significantly positive (negative) if the estimate is positive (negative) and both the sign test and the Wilcoxon Signed Rank Test show that estimate to be significant. Based on the test result we classify the risk aversion profile as being consistent with one of the following utility functions: Friedman and Savage (FS); Friedman and Savage—partial support (FS_P); Markowitz (M); Kahneman-Tversky—partial support, (KT_P); or "Other."

We label the firm "FS" if the estimated risk aversion is positive for a range of wealth, then negative, and then positive again. If the tests detect the presence of only two regions, positive followed by negative, we consider this as partial evidence in support of Friedman and Savage utility function since risk aversion could turn positive again for large wealth levels and we label such firms "FS_P". For all Friedman and Savage patterns (both "FS" and "FS_P"), we test the sign of the estimated risk aversion coefficient at $w = 1$. If risk aversion is not statistically different from zero and risk aversion changes from positive to negative then this provides evidence consistent with Markowitz utility and we label such firms "M."[14] The sign tests also identify risk aversion profiles that provide partial evidence in support of Kahneman-Tversky utility, labeled "KT_P." For these firms risk aversion changes from negative to positive, but the second condition is not met: risk aversion is not zero at $w = 1$. We classify all other risk aversion profiles as "Other." The above classification is repeated using *t*-test to show significance (Table 5). We find that the results are virtually identical to Table 4.

The sign tests only help classify utility functions consisting of convex and concave regions. We require additional tests to identify CARA and CRRA type utility functions. These functions are characterized not only by the sign of the risk aversion estimate but also the shape of the risk aversion function.

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[14] Note that this is a very strong test for Markowitz utility. Given that risk aversion coefficients are statistical estimates with standard error associated with them, it is a lot to require that risk aversion equals to zero exactly at $w = 1$. We develop additional tests that reflect this uncertainty in the next section.

Markowitz Utility Function

To classify estimates of risk aversion as being consistent with Markowitz utility function we need to show that three properties hold. The estimates of risk aversion must be *positive* for wealth levels below one, and the estimates of risk aversion must be *negative* for wealth levels above one. The third assumption is most restrictive: Risk aversion must equal zero in the neighborhood where wealth equals one. The sign tests are conservative tests. These tests identify regions where risk aversion is positive or negative. The defining property of Markowitz utility function is that the estimate of risk aversion crosses the horizontal (wealth) axis exactly at the wealth level of one. When testing a point hypothesis, whether or not risk aversion is zero at the wealth level of one, the sign tests may be two restrictive. We address this by developing additional tests for the behavior of the estimated risk aversion function at the point $w = 1$.

To test whether or not an estimate of risk aversion is significantly different from zero when wealth equals one we develop the following Monte Carlo simulation. There are 101 equally spaced wealth points in the interval [0.95, 1.05]. For a given firm, for each wealth level w_i in this interval, the estimate of risk aversion, $\hat{a}(w_i)$ is a random variable with mean $\bar{a}(w_i)$ and standard deviation $\sigma[\hat{a}(w_i)]$. In simulations, for each wealth level w_i in the interval, the estimate of risk aversion, $\hat{a}(w_i)$ is drawn from $N(\bar{a}_i, \sigma(\hat{a}_i))$, where $\bar{a}_i$ the average risk aversion (for the firm) for wealth level w_i, and $\sigma(\hat{a}_i)$ is the standard deviation. This is done for all 101 levels of wealth. For each draw of risk aversion coefficients, $\{\hat{a}_i\}_{i=1}^{101}$, several polynomial models with wealth as the independent variable are fitted. The objective is to use the fitted model to generate estimates of risk aversion at $w = 1$. For each repetition of the Monte Carlo experiment each polynomial model generates one estimate of the value of risk aversion at $w = 1$. We select the best model (the fit reported in Table 6) and then test whether the estimates of risk aversion generated by the best model are statistically different from zero at $w = 1$. The results of these tests are reported in Table 7. We find evidence supporting Markowitz utility function for several firms: *Alcoa* (AA), *American International Group* (AIG), *AT&T* (T), *Citigroup* (C), *Eastman Kodak* (EK), and *PepsiCo.* (PEP). For *Alcoa* (AA), risk aversion is negative in [0.95, 0.958], positive in [0.967, 0.988], and negative in [1.015, 1.05]. Risk aversion is not statistically different from zero at $w = 1$ (Table 4). This profile is remarkably close to Markowitz utility. For *American International Group* (AIG) risk aversion is positive in [0.95, 0.999], zero at $w = 1$, and negative in [1.007, 1.049], as shown in Table 4. The pattern for *Citigroup* (C) is similar: risk aversion is positive in [0.95, 0.992] and negative in [1.005, 1.049]. For *Eastman Kodak* (EK) risk aversion is positive in [0.95, 0.996], zero at $w = 1$, and negative in [1.004, 1.05]. Table 4 shows that for *PepsiCo.* (PEP) risk aversion is positive in [0.95, 0.999], zero at $w = 1$, and negative in the interval [1.002, 1.05]. For *AT&T* (T) the *t*-test (Table 5) indicates that risk aversion is positive for wealth in the interval [0.95, 0.994] and negative for [1.003, 10.05]. For *Alcoa* (AA), *American International Group* (AIG), *AT&T* (T), *Citigroup* (C), *Eastman Kodak* (EK), and *PepsiCo.* (PEP) in Monte Carlo simulations, risk aversion for $w = 1$ is statistically indistinguishable from zero (Table 6 and Table 7).

Markowitz utility function has a restrictive assumption that risk aversion is zero precisely at wealth level of one. Since we are testing a strong hypothesis, sometimes different tests provide close but different results. In the case of *3M Company* (MMM), the sign test shows that risk aversion is positive in the region [0.95, 0.994] and negative in the region [0.997, 1.05]. According to the univariate test, risk aversion is statistically not different from zero in the interval [0.995, 0.996] and is negative at wealth level of one. Our Monte Carlo tests first fit a model, and then compute estimates of risk aversion at the wealth level of one (Table 6 and Table 7). These tests show that risk aversion is statistically indistinguishable from zero at $w = 1$ (the average estimate is -0.49 with *t*-value of -0.76). We therefore classify MMM as providing partial support for Markowitz utility function.

Similarly to MMM, we classify *Walt Disney Company* (DIS) as providing partial support for Markowitz utility. Risk aversion is positive in [0.95, 1.007] and negative in the region [1.014, 1.05]. According to the sign tests, the estimate of risk aversion is positive at the wealth of one. Monte Carlo tests show that risk aversion estimates at w=1 are statistically not distinguishable from zero, with the average estimate of -0.26 (*t*-statistics equals -0.57).

Another example of partial support for Markowitz utility is *American Express* (AXP). Estimated risk aversion for this firm is *positive* in the region [0.95, 1.004], *negative* for wealth in the interval [1.009, 1.03], and then *positive* again in the region [1.047, 1.05], a pattern consistent with Friedman and Savage utility function. According to the univariate sign tests estimated risk aversion is positive at the wealth level of one. According to the Monte Carlo simulations risk aversion is statistically indistinguishable from zero at the wealth level of one (the average value generated by the best fitting model equals 0.96 with *t*-statistic of 1.47). We interpret this as partial support of Markowitz hypothesis.

Kahneman and Tversky Utility Function

To be classified as a Kahneman-Tversky utility, risk aversion must be *negative* below the wealth level of one, *equal zero* at the wealth level of one, and be *positive* above it. Estimates of risk aversion for *Union Carbide* (UK) are consistent with this profile. According to the univariate sign tests (Table 4), risk aversion is negative for wealth in the interval [0.95, 1.003] and is positive for wealth levels in the interval [1.011, 1.05]. Risk aversion is negative for $w =$

1, but it equals zero for $w = 1.004$. To further investigate the behavior of risk aversion at $w = 1$, we perform Monte Carlo experiments where several models are fit to the estimated risk aversion. The best fitting model is used to generate risk aversion estimates at $w = 1$ (Table 9).[15] The estimates are statistically indistinguishable from zero, consistent with Kahneman-Tversky utility.

For several companies risk aversion estimates provide partial support for the profile implied by Kahneman and Tversky (Prospect Theory) value function. These companies have estimated risk aversion that is negative below the wealth level of one and positive above it. The change, however, does not occur exactly at $w = 1$. We interpret these cases as providing partial support for Kahneman-Tversky's hypothesis.

Consider, for example, *International Business Machines* (IBM). According to the univariate tests (Table 4), risk aversion is negative in the interval [0.95, 1.019] and is positive in the interval [1.029, 1.05]. Risk aversion is negative (significantly different from zero) at the wealth level of one. The Monte Carlo simulation confirms the results from the univariate sign tests.

Caterpillar (CAT) is another company that has a risk aversion profile very close to the one implied by the Prospect Theory. Univariate tests (Table 4) show that there is a region where risk aversion is negative, [0.95, 0.983], followed by a region where risk aversion is positive, [0.987, 1.04]. This pattern is broadly consistent with Kahneman-Tversky utility. Risk aversion, however, becomes negative again for wealth in the interval [1.047, 1.05]. Prospect Theory in its original form does not make a prediction about the shape of the value function for large wealth level. The value function is commonly drawn as a concave function for *all* wealth levels above the current wealth. There is another reason why we cannot classify

[15] These experiments are identical to the experiments used to test for Markowitz property.

CAT as a strong case of the Prospect Theory utility function. Univariate tests show that risk aversion is positive at $w = 1$. When we perform Monte Carlo simulations and use two best fitting models to estimate risk aversion in the neighborhood of $w = 1$, we find estimates to be positive and significant (model fit is reported in Table 8 and estimates are reported in Table 9).

Estimated risk aversion for *Sears Roebuck & Co.* (S) changes from negative for wealth in the interval [0.95, 0.966] to positive in the interval [0.97, 1.05], a pattern consistent with Kahneman-Tversky utility (Table 4 and Table 5). The change in sign, however, does not take place at the wealth level of one. This is confirmed by Monte Carlo experiments.

We conclude that IBM, CAT, S have estimated risk aversion profiles that provide partial support for Kahneman-Tversky utility.

Friedman and Savage Utility

We find remarkably strong support for Friedman and Savage utility function. Estimated risk aversion has a profile consistent with concave-convex-concave utility for 20 out of 41 firms (Figure 4). *Chevron* (CHV), a company with capitalization of $34.5 Billion in 1996, is one example. Risk aversion is positive in [0.95, 0.984], negative in [0.986, 1.019], and then positive again in [1.02, 1.05]. Another example is *General Electric* (GE), the largest firm in the sample. For GE risk aversion is positive-negative-positive. This risk aversion profile describes investors in *Exxon Mobil* (XOM) and *Home Depot* (HD). Perhaps the reason for finding strong support is that Friedman-Savage hypothesis is the most flexible of the three non-concave utility functions. It does not impose point hypotheses as Kahneman-Tversky and Markowitz do for the behavior of the function at the point $w = 1$.

There are also cases that we interpret as "partial support" for Friedman and Savage utility. For example, for *SBC Communications* (SBC) a region where risk aversion is positive is followed by a region where risk aversion is negative. There is no second positive region, however, and we classify SBC as "FS_P". *Goodyear* (GT) is another example. According to the univariate sign tests (Table 4) risk aversion changes from being negative in the interval [0.95, 0.988] to positive in the interval [0.994, 1.028]. The tests based on *t*-statistics reported in Table 5 confirm this pattern. Among other firms classified in this category are: the second largest firm in the sample, *Microsoft* (MSFT), as well as *Coca-Cola* (KO), *Intel* (INTC), *Johnson & Johnson* (JNJ), and *Procter and Gamble* (PG).

Other Types of Utility Functions

We find evidence supporting CARA and CRRA utility functions. We also find risk aversion profiles that cannot be classified within any class of utility functions that we consider.

For two firms, *Altria Group* (MO) and *Boeing Company* (BA) estimates of risk aversion are positive and constant, which is consistent with CARA utility. For *United Technologies Corporation* (UTX) estimated risk aversion is positive for all wealth levels. We find that the model of risk aversion as a function of $(1/w)$ fits well (Table 8), providing evidence consistent with CRRA preferences.

If risk aversion profile does not match one of the five profiles discussed above we classify these companies as "other" in Table 3. This classification applies to *International Paper* (IP), *Merck & Company* (MRK), *Pfizer* (PFE), and *Wal-Mart Stores* (WMT).

Conclusion

We estimate and analyze risk aversion of investors in the *Dow Jones Industrial Average* stocks. The Dow members are large, visible companies. By studying these firms we minimize the impact of transaction costs and liquidity issues and we minimize the impact of information asymmetries. These firms are extensively followed by analysts and information about the companies is widely available to individual investors.

In this environment, we find evidence supporting the existence of five distinctly different utility functions. We show that asset prices reflect non-concave risk preferences with reference points. The support for utility functions with convex regions—as postulated by Kahneman and Tversky, Friedman and Savage, and by Markowitz—is much stronger that the evidence in favor of the standard uniformly risk-averse preferences. Utility functions with convex regions are evident in the data for 34 out of 41 firms in the sample. Only three out of 41 functions are uniformly concave and conform to the commonly modeling assumptions (CARA, CRRA). For four firms risk aversion profile does not fit into any of the above classes. The results for Kahneman and Tversky utility function and Markowitz utility are remarkably strong, considering the restrictive characteristics of these functions. These functions assume that risk aversion changes sign exactly at the current wealth level. We find statistical support for this hypothesis. Asset prices reflect this property.

We are not the first to report evidence in support of utility functions with convex regions, *per se*. The evidence was found before in controlled experiments.

It is notable, however, that the support is so strong when we use market data—prices of individual stocks and options. Kahneman and Tversky, as well as Friedman and Savage, and Markowitz developed their utility functions to explain very general patterns in human behavior. They did not develop these utility functions with a view of explaining specific well-documented patterns in asset prices. We find that these preferences are reflected in prices of *Dow Jones* stocks. And the evidence, taken together, is strong. Together, these three types of utility account for 80% of the market capitalization of the sample stocks.

Our findings provide two challenges for future research in asset pricing. First, the support for commonly used utility functions is relatively weak. Second, the existence of several types of utility functions challenges the representative agent paradigm.

Appendix

Convexity in the Theory of Choice

The idea that choices involving risk can be explained by the maximization of expected utility dates back at least to Daniel Bernoulli's classical analysis of the St. Petersburg paradox. Concave utility functions correspond to economic intuition and have convenient mathematical properties.[16] Concave functions, however, cannot explain gambling and strong evidence that economic agents willingly participate in activities with negative expected return. This motivated one of the first modifications to the concave utility function.

To explain coexistence of gambling and insurance in human behavior Friedman and Savage (1948) propose that an individual's utility of wealth function is composed of two (strictly) concave segments separated by a (strictly) convex segment (Figure 1). Expected utility theory with a non-concave utility function remains the most parsimonious model of human behavior under uncertainty that allows for gambling (Hartley and Farrell 2002).

Markowitz (1952) argues that the Friedman and Savage utility function should be modified so that the inflection point where the concave region turns into the convex region is located exactly at the individual's current wealth. Markowitz (1952) also suggests that the utility of wealth function has three inflection points. The utility function is monotonically increasing but bounded; it is first convex, then concave, then convex, and finally concave. The middle inflection point is defined to be at the current level of wealth. The first inflection point is below, the third inflection point is above, current wealth (Figure 2).

The notion of increasing marginal utility (convexity) causes certain discomfort among the economists. Kwang (1965) suggested a resolution of the problem that is based on the indivisibility of consumption. The assumption that the marginal utility of income is

[16] For the classical treatment of risk aversion see Arrow (1964), Pratt (1964), Arrow (1965), Arrow (1970). See also Rabin and Thaler (2001).

continually diminishing (and the utility function is therefore concave) is derived from the assumption that the expenditure of the consumer is infinitely divisible. Clearly, this is not the case. Kwang (1965) showed that gambling can be consistent with the principles of utility maximization when indivisibility of consumption is introduced. Individuals purchase lottery tickets with payoffs that give them a positive probability of moving to a new consumption level by being able to afford an indivisible consumption good. If the cost of purchasing a car, a house, a university education, or a business, appears far beyond the existing means, it becomes rational for an individual agent to participate in a gambling opportunity that offers a chance of a sufficiently high payoff. Winning such a lottery would bring the individual to a qualitatively new "level" of consumption.

Another paper offers a very attractive explanation for the existence of convex regions in the individual's utility function. Hakansson (1970) starts with the observation that since money is only a means to an end (consumption), the derived utility of wealth is dependent on the utility of consumption and the opportunities for achieving it. Mathematically, the derived utility of wealth function is defined as

$$
\begin{aligned}
J[W(t), t] &\equiv Max\, E_t\left[\sum_{s=t}^{T-1} U(C, s) + B(W_T, T)\right], \\
J[W(T), T] &\equiv B(W_T, T).
\end{aligned}
$$

In this case the terminal date T is assumed known and the utility function is assumed additively separable. In this formulation $U(\bullet)$ is the utility of consumption and $B(\bullet)$ is the utility of bequest. Clearly, the utility of present wealth is influenced by preferences over consumption at each future point in time, utility over bequest, the agent's labor income, future interest rates, the risk and return of the future investment opportunities, and borrowing restrictions. Therefore, the determination of an individual's utility of current wealth requires a model of his total economic decision problem, including the description of the investment opportunity set and restrictions, such as borrowing or short-sale constraints. Hakansson (1970) develops such a model. He begins with risk averse preferences over consumption. He then imposes a borrowing constraint of a reasonable form and finds that the constraint gives rise to a Friedman-Savage utility function of current wealth.

Perhaps the most well-known class of value function is the prospect theory *S*-shaped function suggested by Kahneman and Tversky. Based on their experimental results, Kahneman and Tversky (1979) and Tversky and Kahneman (1992) suggest that the value function is convex in the domain of losses (below the current wealth level) and concave in the domain of gains (above the current wealth). This function has one inflection point located at the current level of wealth.

Does convexity create havoc in our asset prices theories that start with the assumption of concave utility? Not necessarily. Jarrow (1988) studies an economy consisting of an infinite number of assets and shows that the Arbitrage Pricing Theory does not require that agents possess preferences that can be represented by risk-averse expected utility functions. Blackburn and Ukhov (2005) observe that, at the first glance, utility functions of individual investors with convex regions corresponding to risk seeking appear to be in sharp contradiction with the "equity premium puzzle." The puzzle states that the degree of individual aversion to risk must be very high to explain *aggregate* equity returns. Can individual investors exhibit risk seeking behavior and at the same time, *in the aggregate*, demand a high positive rate of return for holding risky assets? Blackburn and Ukhov (2005) resolve this paradox by showing that risk-seeking behavior at the individual level can be consistent with risk-averse behavior at the aggregate level. The authors begin with a model where all agents have a convex utility implying they are risk seekers. The agents face a constraint—they cannot infinitely borrow (or sell short). When agents are heterogeneous with respect to the initial endowment, under perfect competition the economy is risk averse.

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Table 1. Dow Jones Industrial Average Components

Company Name	Ticker	Comments
3M Company	MMM	
Alcoa Inc.	AA	
Allied Signal Inc.	ALD	
Altria Group, Inc.	MO	
American Express Co.	AXP	
American International Group	AIG	
AT&T Corp.	T	
Bethlehem Steel	BS	Trading suspended June 7, 2002. Insufficient data.
Boeing Company	BA	
Caterpillar Inc.	CAT	
Chevron	CHV	
Chevron-Texaco	CVX	Chevron (CHV) and Texaco (TX) merged on 10 Oct-2001 to form Chevron-Texaco (CVX). Estimate CHV, CVX, TX.
Citigroup Incorporated	C	
Coca-Cola	KO	
Du Pont	DD	
Eastman Kodak	EK	
Exxon Mobil Corporation	XOM	
General Electric Company	GE	
General Motors	GM	
Goodyear	GT	
Hewlett-Packard Co.	HPQ	
Home Depot, Inc.	HD	
Honeywell International, Inc.	HON	Honeywell merged with Allied Signal Inc. (ALD) on 30-Nov-99. Estimate pre-merger ALD and HON.
Intel Corporation	INTC	
International Business Machines	IBM	
International Paper	IP	
Johnson & Johnson	JNJ	
J.P. Morgan & Company	JPM	Chase Manhattan Bank and J.P. Morgan merged on 31-Dec-2000 to form J.P. Morgan Chase & Company (JPM). Estimated for pre-merger J.P. Morgan & Company.
McDonald's Corp.	MCD	
Merck & Company, Inc.	MRK	
Microsoft Corporation	MSFT	
PepsiCo	PEP	Not a Dow Component; Included for comparison with Coca Cola.
Pfizer, Inc.	PFE	
Procter and Gamble Co.	PG	
SBC Communications	SBC	
Sears Roebuck & Co.	S	
Texaco	TX	Chevron (CHV) and Texaco (TX) merged on 10-Oct-2001.
Union Carbide	UK	Union Carbide became a subsidiary of Dow Chemical Company on 6-Feb-2001.
United Technologies Corporation	UTX	
Verizon Communications, Inc.	VZ	
Wal-Mart Stores, Inc.	WMT	
Walt Disney Company	DIS	
Westinghouse Electric	WX	Acquired by BNFL, plc. (United Kingdom) in March 1999; Insufficient data.
Woolworth	Z	Changes its name to Venator on 12-Jun-98; then to Foot Locker on 02-Nov-2001. New ticker (FL) starting 31-Mar-2003. Insufficient data.

Table 2. Summary Statistics and Risk and Return Characteristics

Ticker	Market Cap 2-Jan-96 ($MM)	Market Cap 31-Dec-03 ($MM)	Beta S&P500	Beta CRSP VW	Avg. Return Monthly (96-03)	Variance of Return (96-03)	HPR 2-Jan-96 to 31-Dec-03
AA	9,635.79	32,883.49	1.106	1.105	1.73%	1.22%	241.26%
AIG	44,268.34	172,854.66	0.881	0.778	1.35%	0.58%	290.47%
ALD	13,930.51	3,559.22	1.113	1.016	0.98%	1.36%	-74.45%
AXP	20,102.81	62,037.48	1.052	0.985	1.69%	0.70%	208.60%
BA	27,408.79	33,721.10	0.842	0.809	0.63%	0.89%	23.03%
BS	1,590.25	32.75*	1.619	1.564	-2.53%	3.14%	-97.94%
C	19,534.00	250,402.18	1.125	1.051	2.25%	0.97%	1181.88%
CAT	11,752.38	28,661.82	0.894	0.826	1.48%	0.83%	143.88%
CHV	34,486.87	92,347.37	0.730	0.679	1.00%	0.39%	167.78%
DD	39,983.26	45,742.10	0.901	0.849	0.88%	0.59%	14.40%
DIS	31,823.87	47,718.34	1.027	0.967	0.35%	0.81%	49.95%
EK	23,477.09	7,356.35	0.915	0.860	-0.31%	0.89%	-68.67%
GE	122,765.21	311,065.84	0.963	0.906	1.57%	0.59%	153.38%
GM	39,156.10	29,944.10	1.078	1.058	1.01%	1.04%	-23.53%
GT	6,840.30	1,377.94	1.211	1.167	-0.64%	1.65%	-79.86%
HD	22,589.25	80,747.42	0.961	0.977	1.84%	0.91%	257.46%
HON	6,158.43	28,818.36	1.110	1.015	0.93%	1.45%	367.95%
HPQ	42,735.30	70,038.75	1.329	1.411	1.19%	1.84%	63.89%
IBM	50,736.88	159,448.80	1.111	1.064	2.12%	1.16%	214.27%
INTC	48,142.85	207,908.35	1.309	1.293	2.34%	2.08%	331.86%
IP	10,202.16	20,712.93	0.994	0.917	0.67%	0.93%	103.03%
JNJ	54,562.49	153,334.27	1.224	1.184	1.34%	1.12%	181.03%
JPM	15,214.51	74,939.15	0.637	0.550	1.40%	0.98%	392.55%
KO	94,077.15	124,414.23	0.746	0.689	0.81%	0.65%	32.25%
MCD	31,680.69	31,513.34	0.935	0.835	0.52%	0.65%	-0.53%
MMM	28,430.95	66,738.60	0.673	0.649	1.24%	0.46%	134.74%
MO	76,691.37	110,536.38	0.557	0.505	1.73%	0.98%	44.13%
MRK	79,112.18	102,794.86	0.720	0.586	1.24%	0.74%	29.94%
MSFT	52,952.50	295,294.93	1.179	1.146	2.66%	1.63%	457.66%
PEP	43,926.82	80,034.75	0.814	0.756	0.99%	0.55%	82.20%
PFE	39,600.59	269,621.71	0.679	0.587	1.69%	0.58%	580.85%
PG	57,050.77	129,517.09	0.515	0.481	1.34%	0.55%	127.02%
S	15,740.19	11,977.52	0.834	0.791	0.43%	1.14%	-23.90%
SBC	35,600.58	86,309.17	0.882	0.766	0.65%	0.85%	142.44%
T	107,251.90	16,034.42	1.077	1.071	0.23%	1.18%	-85.05%
TX	20,798.66	38,476.89*	0.596	0.524	1.22%	0.46%	85.00%
UK	5,147.29	7,063.99*	0.897	0.819	1.10%	0.86%	37.24%
UTX	11,438.25	44,594.88	0.977	0.920	1.85%	0.79%	289.87%
VZ	29,627.41	96,875.17	0.917	0.789	0.80%	0.83%	226.98%
WMT	53,376.33	229,588.78	0.741	0.663	2.05%	0.71%	330.13%
WX	7,111.34	45,072.06*	0.852	0.916	2.95%	0.87%	533.81%
XOM	100,110.29	271,001.80	0.645	0.610	1.14%	0.25%	170.70%
Z	1,746.16	3,360.76	0.901	0.828	1.70%	2.68%	92.47%

*BS ending price and ending market cap are as of 6/11/2002.
*TX ending price and ending market cap are as of 10/9/2001.
*WX ending price and ending market cap are as of 5/3/2000.
HPR is adjusted for stock splits. All Betas are significant at 1% level or better.

Table 3
Support for Different Utility Functions

Using daily risk aversion estimates from 1996-2004 and wealth levels ranging from 0.95 to 1.05 with step size of 0.001, we test whether the median and mean risk aversion for each wealth is significantly different from zero. Based on whether risk aversion is positive, zero, or negative we determine the risk aversion profile. We then classify risk aversion profile as being consistent with one of the following utility functions: Friedman and Savage, Friedman and Savage, partial support, Markowitz (M), Kahneman-Tversky, partial support, CARA, CRRA, or "Other." If the risk aversion is positive-negative-positive than we classify is as Friedman and Savage. If only a part of this profile is present (positive-negative) then we classify it as FS, Partial support. Companies in bold are discussed in detail in the text.

Friedman and Savage	**Chevron (CHV)**
	Exxon Mobil Corporation (XOM)
	General Electric (GE)
	Home Depot (HD)
	Honeywell International, Inc. (HON)
	McDonald's Corp. (MCD)
Friedman and Savage	Allied Signal Inc. (ALD)
Partial support	**Coca-Cola (KO)**
	Chevron-Texaco (CVX)
	Du Pont (DD)
	General Motors (GM)
	Goodyear (GT)
	Hewlett-Packard Co. (HPQ)
	Intel Corporation (INTC)
	J.P. Morgan & Company (JPM)
	Johnson & Johnson (JNJ)
	Microsoft Corporation (MSFT)
	Procter & Gamble (PG)
	SBC Communications (SBC)
	Texaco (TX)
	Verizon Communications, Inc. (VZ)
Markowitz	**3M Company (MMM)** (RA = 0 for $0.995 \leq w \leq 0.996$)
	Alcoa Inc. (AA) (RA = 0 for $w = 1$)
	American Express Co. (AXP) (RA = 0 for $1.005 \leq w \leq 1.008$)
	American International Group (AIG) (RA = 0 for $w = 1$)
	AT&T (T) (RA = 0 for $w = 1$)
	Citigroup Incorporated (C) (RA = 0 for $w = 1$)
	Eastman Kodak (EK) (RA = 0 for $w = 1$)
	PepsiCo. (PEP) (RA = 0 for $w = 1$)
	Walt Disney Company (DIS) (RA = 0 for $1.008 \leq w \leq 1.014$)
Kahneman and Tversky	**Caterpillar Inc. (CAT) (Partial support)**
	International Business Machines (IBM) (Partial support)
	Sears Roebuck & Co. (S) (Partial support)
	Union Carbide (UK)
CARA	**Altria Group, Inc. (MO)**
	Boeing Company (BA)
CRRA	**United Technologies Corporation (UTX)**
Other	**International Paper (IP)**
	Merck & Company, Inc. (MRK)
	Pfizer, Inc. (PFE)
	Wal-Mart Stores (WMT)

Table 4
Univariate Sign Test
Using daily risk aversion estimates from 1996-2004 and wealth levels ranging from 0.95 to 1.05 with step size of 0.001, we test whether the median and mean risk aversion for each wealth is significantly different from zero. We use a sign test to determine if the median is significantly different from zero and we use the Wilcoxon Signed Rank Test to measure whether the mean is significantly different from zero. We classify the risk aversion for a particular wealth level as significantly positive (negative) if the estimate is positive (negative) and both the sign test and the Wilcoxon Signed Rank Test show that the estimate to be significant. Based on the test results we classify risk aversion profile as being consistent with one of the following utility functions: Friedman and Savage (FS), Friedman and Savage, partial support, (FS_P), Markowitz (M), Kahneman-Tversky, partial support, (KT_P), or "Other."

Ticker	Sign of RA *Wealth*	Sign of RA *Wealth*	Sign of RA *Wealth*	Sign of RA *Wealth*	RA at W=1	Utility
AA	Negative *0.95-0.958*	Positive *0.967-0.988*	Negative *1.015-1.05*		W=1 *Insignificant*	M
AIG	Positive *0.95-0.999*	Negative *1.007-1.049*			W=1 *Insignificant*	M
ALD	Positive *0.95-1.043*	Negative *1.049-1.05*			W=1 *Positive*	FS_P
AXP	Positive *0.95-1.004*	Negative *1.009-1.03*	Positive *1.047-1.05*		W=1 *Positive*	FS
BA	Positive *0.95-0.97*	Positive *0.99-1.047*			W=1 *Positive*	Other
CAT	Negative *0.95-0.983*	Positive *0.987-1.04*	Negative *1.047-1.05*		W=1 *Positive*	Other
CHV	Positive *0.95-0.984*	Negative *0.986-1.019*	Positive *1.02-1.05*		W=1 *Negative*	FS
CITI	Positive *0.95-0.992*	Negative *1.005-1.049*			W=1 *Insignificant*	M
CVX	Positive *0.95-0.962*	Negative *0.967-1.05*			W=1 *Negative*	FS_P
DD	Positive *0.95-0.952*	Negative *0.97-0.995*	Negative *1.007-1.05*		W=1 *Insignificant*	FS_P
DIS	Positive *0.95-1.007*	Negative *1.014-1.05*			W=1 *Positive*	FS_P
EK	Positive *0.95-0.996*	Negative *1.004-1.05*			W=1 *Insignificant*	M

Table 4 (Continued)
Univariate Sign Test

Ticker	Sign of RA ***Wealth***	Sign of RA ***Wealth***	Sign of RA ***Wealth***	Sign of RA ***Wealth***	RA at W=1	Utility
GE	Positive *0.95-0.977*	Negative *0.979-1.039*	Positive *1.048-1.05*		W=1 *Negative*	FS
GM	Positive *0.95-0.953*	Negative *0.955-1.049*			W=1 *Negative*	FS_P
GT	Positive *0.95-0.988*	Negative *0.994-1.028*			W=1 *Negative*	FS_P
HD	Positive *0.95-0.961*	Negative *0.966-1.031*	Positive *1.035-1.05*		W=1 *Negative*	FS
HON	Positive *0.95-0.978*	Negative *0.981-1.012*	Positive *1.018-1.037*	Negative *1.043-1.05*	W=1 *Negative*	FS
HPQ	Positive *0.95-0.986*	Negative *0.994-1.05*			W=1 *Negative*	FS_P
IBM	Negative *0.95-1.019*	Positive *1.029-1.05*			W=1 *Negative*	Other
IP	Negative *0.95-0.952*	Positive *0.97-1.005*	Negative *1.011-1.05*		W=1 *Positive*	Other
INTC	Positive *0.95-0.97*	Negative *0.987-1.05*			W=1 *Negative*	FS_P
JNJ	Positive *0.95-0.967*	Negative *0.968-1.046*			W=1 *Negative*	FS_P
JPM	Positive *0.95-1.019*	Negative *1.021-1.05*			W=1 *Positive*	FS_P
KO	Positive *0.95-1.012*	Negative *1.023-1.05*			W=1 *Positive*	FS_P
MCD	Positive *0.95-0.958*	Negative *0.966-0.971*	Positive *0.985-1.049*		W=1 *Positive*	FS
MMM	Positive *0.95-0.994*	Negative *0.997-1.05*			W=1 *Negative*	FS_P
MO	Positive *0.95-1.05*				W=1 *Positive*	Other
MRK	Negative *0.95-1.026*	Positive *1.038-1.05*			W=1 *Negative*	Other

Table 4 (Continued)
Univariate Sign Test

Ticker	Sign of RA ***Wealth***	Sign of RA ***Wealth***	Sign of RA ***Wealth***	Sign of RA ***Wealth***	RA at W=1	Utility
MSFT	Positive *0.95-0.953*	Negative *0.967-1.05*			W=1 *Negative*	FS_P
PEP	Positive *0.95-0.999*	Negative *1.002-1.05*			W=1 *Insignificant*	M
PFE	Negative *0.95-0.962*	Positive *0.964-1.023*	Negative *1.025-1.05*		W=1 *Positive*	Other
PG	Positive *0.95-0.985*	Negative *0.987-1.044*			W=1 *Negative*	FS_P
S	Negative *0.95-0.966*	Positive *0.97-1.05*			W=1 *Positive*	Other
SBC	Positive *0.952-0.957*	Negative *0.993-1.002*	Negative *1.007-1.036*		W=1 *Negative*	FS_P
T	Negative *0.999-1.05*				W=1 *Negative*	Other
TX	Positive *0.95-0.991*	Negative *0.995-1.05*			W=1 *Negative*	FS_P
UK	Negative *0.95-1.003*	Positive *1.011-1.05*			W=1 *Negative*	KT_P
UTX	Positive *0.95-1.05*				W=1 *Positive*	Other
VZ	Positive *0.95-0.97*	Negative *0.972-1.05*			W=1 *Negative*	FS_P
WMT	Negative *0.95-0.96*	Negative *0.982-1.029*	Positive *1.048-1.05*		W=1 *Negative*	Other
WX	Positive *0.95-0.973*	Negative *0.986-1.05*			W=1 *Negative*	FS_P
XOM	Positive *0.95-0.976*	Negative *0.98-1.023*	Positive *1.034-1.047*		W=1 *Negative*	FS
Z	Negative *0.95-1.05*				W=1 *Negative*	Other

Table 5
Univariate t-test

Using daily risk aversion estimates from 1996-2003 and wealth levels ranging from 0.95 to 1.05 with step size of 0.001, we test whether the median and mean risk aversion for each wealth is significantly different from zero. A t-test is used to determine significance. The t-statistic is $t = \bar{a}/(s/\sqrt{n})$, where $\bar{a}$ is the sample average risk aversion for a firm, s is the estimated standard deviation, and n is the number of observations. The mean is considered significant for a particular wealth level if it is significant at the 10% level. Based on the test results we classify risk aversion profile as being consistent with one of the following utility functions: Friedman and Savage (FS), Friedman and Savage, partial support, (FS_P), Markowitz (M), Kahneman-Tversky, partial support, (KT_P), or "Other."

Ticker	**Sign of RA** ***Wealth***	**Sign of RA** ***Wealth***	**Sign of RA** ***Wealth***	**Sign of RA** ***Wealth***	**RA at W=1**	**Utility**
AA	Negative *0.95-0.961*	Positive *0.965-1.012*	Negative *1.015-1.05*		W=1 *Positive*	M_P
AIG	Positive *0.95-0.998*	Negative *1.001-1.05*			W=1 *Insignificant*	M
ALD	Positive *0.95-1.041*	Negative *1.046-1.05*			W=1 *Positive*	FS_P
AXP	Positive *0.95-1.005*	Negative *1.007-1.044*			W=1 *Positive*	FS_P
BA	Positive *0.95-0.968*	Negative *0.974-0.977*	Positive *0.986-1.048*		W=1 *Positive*	FS
CAT	Negative *0.95-0.985*	Positive *0.987-1.042*	Negative *1.047-1.05*		W=1 *Positive*	Other
CHV	Positive *0.95-0.984*	Negative *0.986-1.019*	Positive *1.02-1.05*		W=1 *Negative*	FS
CITI	Positive *0.95-0.994*	Negative *0.999-1.05*			W=1 *Negative*	M_P
CVX	Positive *0.95-0.962*	Negative *0.965-1.05*			W=1 *Negative*	FS_P
DD	Positive *0.95-0.952*	Negative *0.962-1.049*			W=1 *Negative*	FS_P
DIS	Positive *0.95-1.006*	Negative *1.01-1.05*			W=1 *Positive*	FS_P

Table 5 (Continued)
Univariate t-test

Ticker	Sign of RA ***Wealth***	Sign of RA ***Wealth***	Sign of RA ***Wealth***	Sign of RA ***Wealth***	RA at W=1	Utility
EK	Positive *0.95-0.995*	Negative *0.998-1.05*			W=1 *Negative*	M_P
GE	Positive *0.95-0.977*	Negative *0.979-1.046*			W=1 *Negative*	FS_P
GM	Positive *0.95-0.953*	Negative *0.955-1.05*			W=1 *Negative*	FS_P
GT	Positive *0.95-0.988*	Negative *0.992-1.03*			W=1 *Negative*	FS_P
HD	Positive *0.95-0.961*	Negative *0.963-1.033*	Positive *1.035-1.05*		W=1 *Negative*	FS
HON	Positive *0.95-0.979*	Negative *0.981-1.015*	Positive *1.02-1.037*	Negative *1.041-1.05*	W=1 *Negative*	FS
HPQ	Positive 0.95-0.986	Negative 0.992-1.05			W=1 *Negative*	FS_P
IBM	Negative *0.95-1.017*	Positive *1.025-1.05*			W=1 *Negative*	Other
IP	Negative *0.95-0.957*	Positive *0.971-1.005*	Negative *1.012-1.05*		W=1 *Positive*	Other
INTC	Positive *0.95-0.968*	Negative *0.974-1.05*			W=1 *Negative*	FS_P
JNJ	Positive *0.95-0.965*	Negative *0.967-1.049*			W=1 *Negative*	FS_P
JPM	Positive *0.95-1.019*	Negative *1.021-1.05*			W=1 *Positive*	FS_P
KO	Positive *0.95-1.011*	Negative *1.016-1.05*			W=1 *Positive*	FS_P
MCD	Positive *0.95-0.956*	Negative *0.96-0.981*	Positive *0.986-1.05*		W=1 *Positive*	FS
MMM	Positive *0.95-0.993*	Negative *0.997-1.05*			W=1 *Negative*	FS_P

Table 5 (Continued)
Univariate t-test

Ticker	Sign of RA *Wealth*	Sign of RA *Wealth*	Sign of RA *Wealth*	Sign of RA *Wealth*	RA at W=1	Utility
MO	Positive *0.95-1.05*				W=1 *Positive*	Other
MRK	Negative *0.95-1.026*	Positive *1.035-1.05*			W=1 *Negative*	Other
MSFT	Positive *0.95-0.955*	Negative 0.959-1.05			W=1 *Negative*	FS_P
PEP	Positive *0.95-1.00*	Negative *1.002-1.05*			W=1 *Positive*	M_P
PFE	Negative *0.95-0.962*	Positive *0.964-1.023*	Negative *1.024-1.05*		W=1 *Positive*	Other
PG	Positive *0.95-0.984*	Negative *0.986-1.044*			W=1 *Negative*	FS_P
S	Negative *0.95-0.966*	Positive *0.97-1.05*			W=1 *Positive*	Other
SBC	Positive *0.952-0.982*	Negative *0.993-1.00*	Negative *1.008-1.036*		W=1 *Negative*	FS_P
T	Positive *0.95-0.994*	Negative *1.003-1.05*			W=1 *Insignificant*	M
TX	Positive *0.95-0.99*	Negative *0.993-1.05*			W=1 *Negative*	FS_P
UK	Negative *0.95-1.006*	Positive *1.012-1.05*			W=1 *Negative*	KT_P
UTX	Positive *0.95-1.05*				W=1 *Positive*	Other
VZ	Positive *0.95-0.97*	Negative *0.972-1.05*			W=1 *Negative*	FS_P
WMT	Negative *0.95-0.969*	Negative *0.983-1.045*	Positive *1.048-1.05*		W=1 *Negative*	Other
WX	Positive *0.95-0.975*	Negative *0.984-1.05*			W=1 *Negative*	FS_P
XOM	Positive *0.95-0.978*	Negative *0.982-1.03*	Positive *1.035-1.046*		W=1 *Negative*	FS
Z	Negative *0.95-1.05*				W=1 *Negative*	Other

Table 6
Markowitz Utility Function: Monte Carlo Model Selection

The table reports the results of Monte Carlo model selection tests. There are 101 equally spaced wealth points in the interval [0.95, 1.05]. For each wealth level in the interval, w_i, estimate of risk aversion, $\hat{a}(w_i)$ is drawn from $N(\bar{a}_i, \sigma(\hat{a}_i))$, where $\bar{a}_i$ the average risk aversion (for the firm) for wealth level w_i, and $\sigma(\hat{a}_i)$ is the standard deviation. For each draw of risk aversion coefficients, $\{\hat{a}_i\}_{i=1}^{101}$, several polynomial models with wealth as the independent variable are fitted. The table reports the averages and Monte Carlo standard errors (in parentheses) of the regression coefficients obtained in simulations. The symbols *, **, *** indicate significance at 10%, 5%, and 1% levels, respectively.

Symbol	**Intercept**	**w**	**w²**	**w³**	**AdjR²**
AA	48.87 ***	-48.74 ***			0.04
	(16.13)	(16.03)			(0.03)
	-1283.2 ***	1951.37 ***		-666.36 ***	0.08
	(532.04)	(800.17)		(267.12)	(0.05)
	-630.59 ***		1943.96 ***	-1311.6 ***	0.08
	(265.37)		(799.77)	(533.40)	(0.05)
AIG	151.85 ***	-150.10 ***			0.26
	(26.49)	(26.78)			(0.08)
	1504.07 ***	-2180.5 ***		676.46 **	0.29
	(664.39)	(1001.04)		(336.07)	(0.07)
	781.28 ***		-2191.52 ***	1410.26 ***	0.29
	(331.88)		(1003.9)	(671.43)	(0.07)
AXP	102.55 ***	-100.63 ***			0.25
	(17.83)	(18.08)			(0.07)
	449.81 ***		-1244.74 **	795.88 **	0.27
	(202.26)		(612.57)	(409.97)	(0.07)
	-49023.4 **	148550.00 **	-149848.0 **	50323.20 **	0.30
	(24271.70)	(73092.90)	(73337.4)	(24516.00)	(0.07)
C	43.76 ***	-44.00 ***			0.07
	(15.85)	(15.81)			(0.05)
	14.42 ***			-14.63 ***	0.07
	(5.33)			(5.27)	(0.05)
DIS	71.42 ***	-71.74 ***			0.17
	(16.35)	(16.41)			(0.07)
	23.84 ***			-24.10 ***	0.17
	(5.44)			(5.48)	(0.07)
EK	100.52 ***	-100.41 ***			0.19
	(20.44)	(20.48)			(0.07)
	33.53 ***			-33.34 ***	0.19
	(6.83)			(6.84)	(0.07)
HPQ	14.57 ***			-15.18 ***	0.12
	(4.22)			(4.06)	(0.06)
MMM	162.90 ***	-163.53 ***			0.35
	(22.75)	(22.45)			(0.07)
	53.74 ***			-54.23 ***	0.35
	(7.77)			(7.44)	(0.07)
PEP	178.25 ***	-179.19 ***			0.30
	(23.54)	(24.23)			(0.06)
	58.93 ***			-59.73 ***	0.30
	(8.03)			(7.82)	(0.06)
	-115521 ***	346957 ***	-346995 ***	115559 ***	0.36
	(38071.2)	(114212)	(114154)	(38013.3)	(0.07)
T	29.53 ***			-30.77 ***	0.06
	(12.49)			(12.37)	(0.05)

Table 7
Testing for Markowitz property

The table reports tests of Markowitz property (risk aversion equals zero at w = 1) based on Monte Carlo simulations. There are 101 equally spaced wealth points in the interval [0.95, 1.05]. For each wealth level in the interval, w_i, estimate of risk aversion, $\hat{a}(w_i)$ is drawn from $N(\overline{a}_i, \sigma(\hat{a}_i))$, where $\overline{a}_i$ the average risk aversion (for the firm) for wealth level w_i, and $\sigma(\hat{a}_i)$ is the standard deviation. For each draw of risk aversion coefficients, $\{\hat{a}_i\}_{i=1}^{101}$, several polynomial models with wealth as the independent variable are fitted. For the best fitting models we compute the value of risk aversion at w = 1 and test whether risk aversion equals to zero.

Symbol	Model	Average	St. Dev.	*t*-value
AA	$\beta_0 + \beta_1 \cdot w$	0.13	0.67	0.19
	$\beta_0 + \beta_1 \cdot w + \beta_2 \cdot w^3$	1.83	1.21	1.52
	$\beta_0 + \beta_1 \cdot w^2 + \beta_2 \cdot w^3$	1.82	1.21	1.51
AIG	$\beta_0 + \beta_1 \cdot w + \beta_2 \cdot w^3$	0.02	1.03	0.02
	$\beta_0 + \beta_1 \cdot w^2 + \beta_2 \cdot w^3$	0.01	1.03	0.01
AXP	$\beta_0 + \beta_1 \cdot w^2 + \beta_2 \cdot w^3$	0.94	0.66	1.43
	$\beta_0 + \beta_1 \cdot w + \beta_2 \cdot w^2 + \beta_3 \cdot w^3$	0.96	0.66	1.47
C	$\beta_0 + \beta_1 \cdot w$	-0.24	0.46	-0.52
	$\beta_0 + \beta_1 \cdot w^3$	-0.21	0.46	-0.44
DIS	$\beta_0 + \beta_1 \cdot w$	-0.32	0.46	-0.70
	$\beta_0 + \beta_1 \cdot w^3$	-0.26	0.46	-0.57
EK	$\beta_0 + \beta_1 \cdot w$	0.11	0.59	0.18
	$\beta_0 + \beta_1 \cdot w^3$	0.19	0.59	0.33
HPQ	$\beta_0 + \beta_1 \cdot w^3$	-0.61	0.36	**-1.69**
MMM	$\beta_0 + \beta_1 \cdot w$	-0.63	0.64	-0.99
	$\beta_0 + \beta_1 \cdot w^3$	-0.49	0.65	-0.76
PEP	$\beta_0 + \beta_1 \cdot w$	-0.94	0.75	-1.25
	$\beta_0 + \beta_1 \cdot w^3$	-0.79	0.76	-1.04
	$\beta_0 + \beta_1 \cdot w + \beta_2 \cdot w^2 + \beta_3 \cdot w^3$	-0.67	1.20	-0.56
T	$\beta_0 + \beta_1 \cdot w^3$	-1.25	1.07	-1.16

Table 8
Monte Carlo Model Selection: Other Types of Utility Functions
The table reports Monte Carlo model selection experiments for utility functions of firms that are classified as "other" based on univariate tests. The "other" category includes all risk aversion profiles that are not classified as (a) Friedman and Savage, (b) Markowitz, or (c) Kahneman and Tversky. The symbols *, **, *** indicate significance at 10%, 5%, and 1% levels, respectively.

	Intercept	**w**	**w^2**	**w^3**	**(1/w)**	**AdjR²**
CAT	-1161.05 ***		3395.71 ***	-2231.55 ***		0.15
	(347.27)		(1040.04)	(691.61)		(0.06)
	92823.4 ***	-282200 ***	285699 ***	-96318.8 ***		0.18
	(37002.7)	(111352.0)	(111641.0)	(37291.3)		(0.07)
IBM	-45.32 ***			41.19 ***		0.27
	(6.32)			(6.31)		(0.07)
	65495.9 ***	-196389 ***	196055 ***	-65165.6 ***		0.31
	(27961.3)	(83997.5)	(84070.5)	(28034.2)		(0.07)
S	-2305.67 *	4554.66 *	-2244.43 *			0.06
	(1371.62)	(2740.93)	(1368.46)			(0.05)
	-785.60 *		2302.32 *	-1512.18 *		0.06
	(458.48)		(1372.2)	(912.82)		(0.05)
UK	-34.22 ***			34.29 ***		0.05
	(15.87)			(15.59)		(0.04)
UTX	-45.65 ***				50.72 ***	0.041
[CRRA]	(21.95)				(21.97)	(0.037)

Table 9
Testing the Behavior of Other Type of Utility Functions
The table reports Monte Carlo model selection experiments for utility functions of firms that are classified as "other" based on univariate tests. The "other" category includes all risk aversion profiles that are not classified as (a) Friedman and Savage, (b) Markowitz, or (c) Kahneman and Tversky. For CAT, IBM, S, and UK we test the null hypothesis that risk aversion equals zero at $w = 1$ (as implied by Kahneman-Tversky utility).

	Model	**Average**	**St. Dev.**	***t*-value**
CAT At $w = 1$	$\beta_0 + \beta_1 \cdot w^2 + \beta_2 \cdot w^3$	3.12	1.47	2.12
	$\beta_0 + \beta_1 \cdot w + \beta_2 \cdot w^2 + \beta_3 \cdot w^3$	3.08	1.47	2.10
IBM At $w = 1$	$\beta_0 + \beta_1 \cdot w^3$	-4.13	0.56	-7.42
	$\beta_0 + \beta_1 \cdot w + \beta_2 \cdot w^2 + \beta_3 \cdot w^3$	-4.50	0.82	-5.46
S At $w = 1$	$\beta_0 + \beta_1 \cdot w + \beta_2 \cdot w^2$	4.56	1.48	3.07
	$\beta_0 + \beta_1 \cdot w + \beta_2 \cdot w^2 + \beta_3 \cdot w^3$	4.55	1.48	3.07
UK At $w = 1$	$\beta_0 + \beta_1 \cdot w^3$	0.07	1.37	0.05

Figure 1. Friedman and Savage Utility of Wealth Function.
Utility as a function of wealth. The function has two concave regions separated by a convex region.

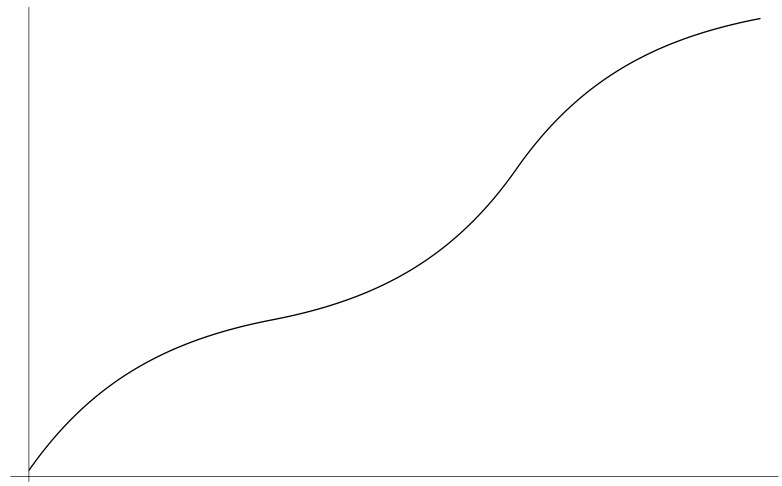

Figure 2. Markowitz Utility of Wealth Function.
Utility as a function of wealth. The function has three inflection points. The second inflection point, where the function changes from being concave to being convex, is located at the current level of wealth.

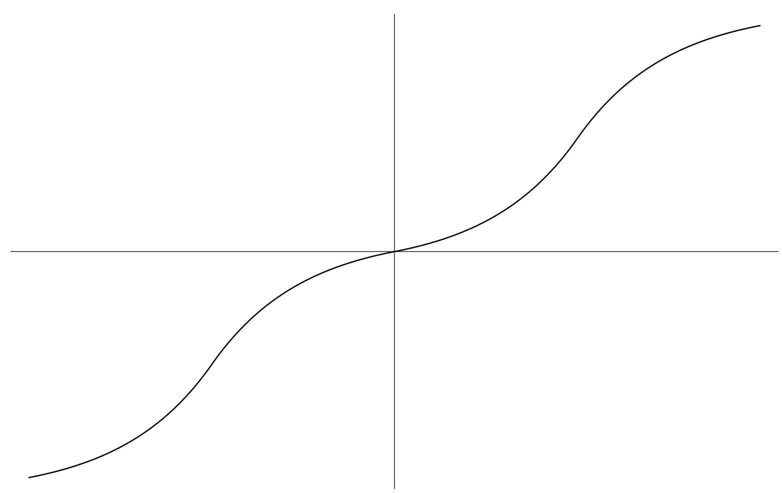

Figure 3. Risk Aversion Consistent With Markowitz Utility.
Risk aversion as a function of wealth for investors in *3MCompany* (MMM), *Eastman Kodak* (EK), *Walt Disney* (DIS), and *PepsiCo* (PEP). The shape of the function is consistent with Markowitz utility function that is concave in the neighborhood of wealth below 1 (positive risk aversion), convex in the neighborhood of wealth above 1 (negative risk aversion), and has an inflection point at 1 (risk aversion changes from positive to negative). Estimated level of risk aversion and a standard error band are shown.

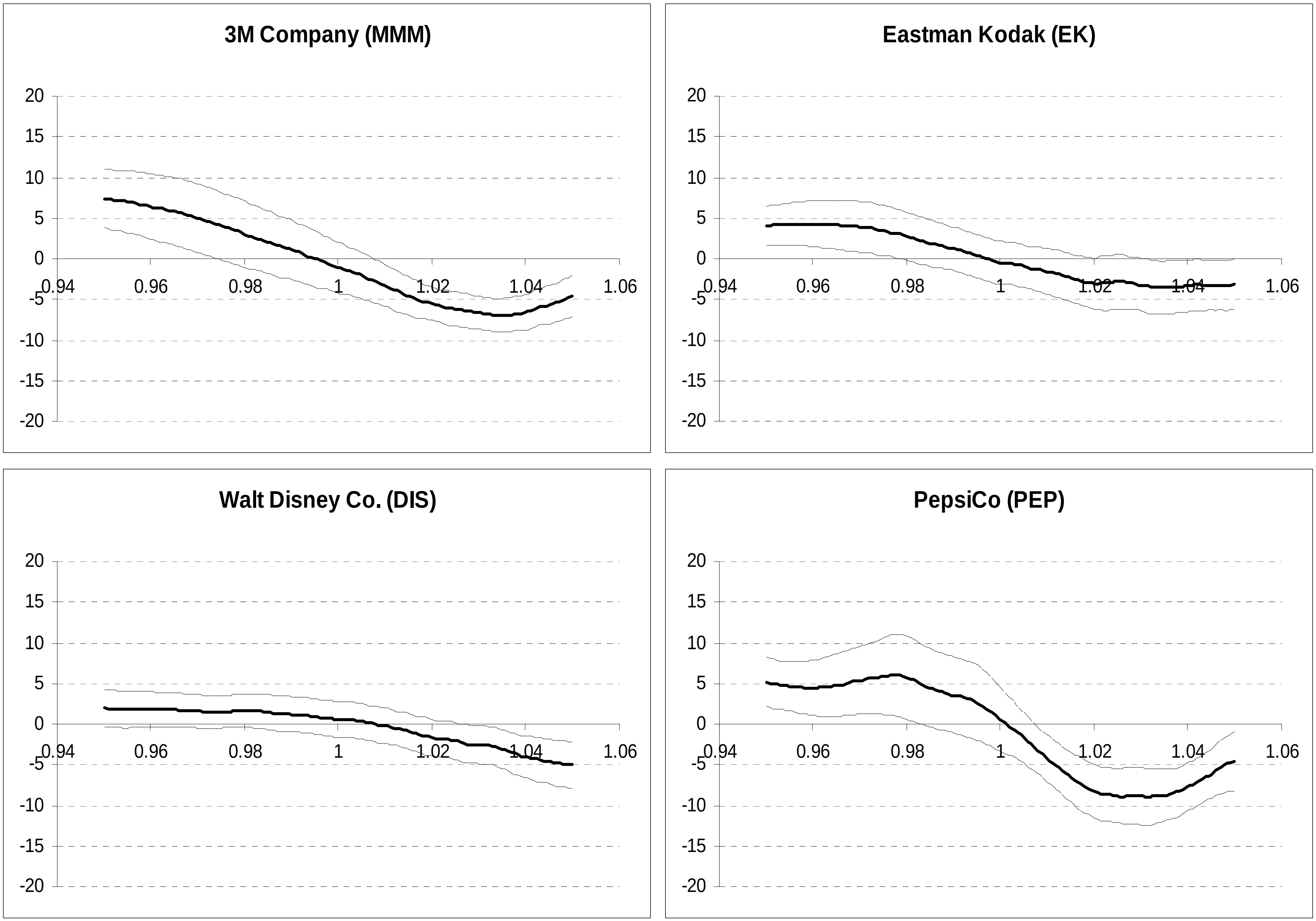

Figure 4. Risk Aversion Consistent With Friedman-Savage Utility.
Risk aversion as a function of wealth for investors in *Chevron* (CHV), *General Electric* (GE), *General Motors* (GM), and *Procter and Gamble* (PG). The shape of the function is consistent with Friedman and Savage utility function that is concave (positive risk aversion), convex (negative risk aversion), and then concave again (positive risk aversion). Estimated level of risk aversion and a standard error band are shown.